Corrections to Scaling and Critical Amplitudes in SU(2) Lattice Gauge Theory *

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We calculate the critical amplitudes of the Polyakov loop and its susceptibility at the deconfinement transition of $SU(2)$ gauge theory. To this end we carefully study the corrections to the scaling functions of the observables coming from irrelevant exponents. As a guiding line for determining the critical amplitudes we use envelope equations derived from the finite size scaling formulae for the observables. The equations are then evaluated with new high precision data obtained on $N_\sigma^3 \times 4$ lattices for $N_\sigma = 12, 18, 26$ and 36. We find different correction-to-scaling behaviours above and below the transition. Our result for the universal ratio of the susceptibility amplitudes is $C_+/C_- = 4.72(11)$ and agrees perfectly with a recent measurement for the 3d Ising model.

1. INTRODUCTION

In systems which exhibit a second order transition in the thermodynamic limit ($V \to \infty$) the critical observables behave as

$$O_\infty = a_0 |t|^{-\rho} \text{ for } |t| \to 0. \tag{1}$$

The coefficient a_0 is the critical amplitude, ρ the critical exponent of the observable O. The variable t is the reduced temperature $t = (T - T_c)/T_c$. In particular, the correlation length ξ, the magnetization or order parameter $\langle M \rangle$ and its susceptibility χ behave for zero external magnetic field H close to the transition as follows

$$\begin{aligned} \xi &= f_\pm |t|^{-\nu} , & (2)\\ \langle M \rangle &= B(-t)^\beta \text{ for } t < 0 , & (3)\\ \chi &= C_\pm |t|^{-\gamma} . & (4) \end{aligned}$$

The index on the amplitude refers to the symmetric (+) or to the broken phase (−).

Certain ratios of critical amplitudes, for example C_+/C_- or f_+/f_- are universal [1], like the critical exponents, that is for systems of the same universality class they are equal. The 3d Ising model and $SU(2)$ gauge theory in (3+1) dimensions are in one class. In the Ising model such ratios have been calculated [2] using very large lattices.

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The aim of this paper is the calculation of the critical amplitudes B, C_+ and C_- for $SU(2)$ with data from moderate size lattices using finite size scaling techniques.

2. FINITE SIZE SCALING

In volumes V with a characteristic length scale $L = V^{1/d}$ finite size scaling theory predicts the following scaling form for small $|t|$

$$O(t, L) = L^{\rho/\nu} \cdot Q(tL^{1/\nu}, L^{-\omega}) . \tag{5}$$

Here, $H = 0$ and only the largest irrelevant exponent $\lambda = -\omega$ was taken into account.

The functions $O(t, L)$ build a family of curves, parametrized by L. We calculate the envelope function to this family. At least the leading term in t should coincide with the limiting form (1).

The scaling function $Q(x, y)$ depends on the scaled reduced temperature $x = tL^{1/\nu}$ and the correction-to-scaling variable $y = L^{-\omega}$. We assume a linear dependence of Q on y

$$Q(x, y) = Q_0(x) + yQ_1(x) . \tag{6}$$

The envelope function O_e is then

$$O_e = (\frac{t}{x_0})^{-\rho} \left\{ Q_0 + (\frac{t}{x_0})^{\omega\nu} Q_1 + O(|t|^{2\omega\nu}) \right\} . \tag{7}$$

Here the Q_i have to be taken at x_0, where x_0 is a zero of the function F_O

$$F_O = \rho Q_0(x) + x Q_0'(x) . \tag{8}$$

Comparing the eqs. (1) and (7) we find

$$a_0 = |x_0|^\rho Q_0(x_0)\ . \tag{9}$$

This result is not a surprise. Suppose, there are no corrections to scaling, so that

$$O(t,L) = L^{\rho/\nu} Q_0(x) = |t|^{-\rho}|x|^\rho Q_0(x)\ , \tag{10}$$

and in the thermodynamic limit

$$O_\infty = |t|^{-\rho} \lim_{x\to\infty} |x|^\rho Q_0(x)\ . \tag{11}$$

We consider therefore the amplitude function A_0

$$A_0(x) = |x|^\rho Q_0(x)\ , \tag{12}$$

and its approach to asymptopia. Its derivative is given by

$$A_0'(x) = \mathrm{sign}(x)|x|^{\rho-1} F_O(x)\ . \tag{13}$$

The function F_O is zero at the point x_0 and also if $Q_0(x) \sim |x|^{-\rho}$, that is when Q_0 has reached its asymptotic form. Then $A_0(x)$ attains an extreme value, the critical point amplitude a_0.

From the above it is clear how to proceed: We measure the observable O for several fixed L and calculate the scaling function $Q = L^{-\rho/\nu}O$. Then we determine Q_0 and Q_1 by a linear fit in y of Q at fixed x and control the approach to the asymptotic scaling form by calculating $F_O(x)$. When F_O stays essentially zero, we evaluate $A_0(x) = a_0$. In addition the correction-to-scaling amplitude may be estimated from

$$a_1 = |x|^{-\omega\nu} Q_1(x)/Q_0(x). \tag{14}$$

3. DATA FOR $SU(2)$ GAUGE THEORY

We use the standard Wilson action and work on $N_\sigma^3 \times 4$ lattices. Taking the lattice spacing $a = 1$ makes N_σ equivalent to $L = N_\sigma a$. We have produced four complete new sets of data for $N_\sigma = 12, 18, 26$ and 36 at 74,49,29 and 26 different couplings, respectively. Our updates consisted of one heatbath and two overelaxation steps, we measured every fifth update. The number of measurements was between 20000 and 80000 at each coupling. We have calculated three observables: the Polyakov loop, corresponding to the magnetization, the susceptibility in the broken phase $\chi = V(\langle M^2\rangle - \langle|M|\rangle^2)$, and that for the symmetric phase $\chi_v = V\langle M^2\rangle$. Their amplitudes are B, C_- and C_+.

4. SCALING ANALYSIS OF THE DATA

As input of our analysis we use the same set of critical exponents as [2]

$$\beta = 0.327\ ,\quad \gamma = 1.239\ ,\quad \nu = 0.631\ . \tag{15}$$

The critical coupling was determined in [3] to $4/g_c^2 = 2.29895(10)$. An analysis with our new data fully confirms this result. Moreover it is essentially independent of ω for $\omega = 1.1 - 1.3$.

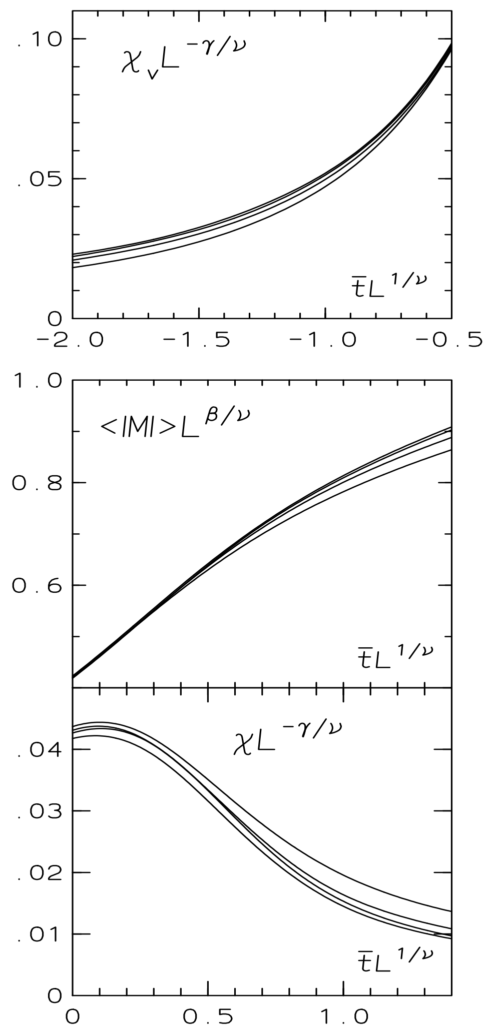

Figure 1. The functions $Q = OL^{-\rho/\nu}$ for $L = 12, 18, 26$ and 36. For Q_M and Q_{χ_v} the lowest curve is the one for $L = 12$, for Q_χ the order is opposite.

In $SU(2)$ the reduced temperature is approximated by $\bar{t} = (4/g^2 - 4/g_c^2)/4/g_c^2$. The symmetric phase occurs at $\bar{t} < 0$, just opposite to magnetic systems. Fig. 1 shows the scaling functions Q. A consistent succession of curves at fixed x for different L emerged only after using very high statistics and many couplings. There is a remarkable difference in the correction-to-scaling behaviours in the two phases $\bar{t} < 0$ and $\bar{t} > 0$. In the symmetric phase, here for χ_v, the correction-to-scaling contribution is linear in y, the sign of a_1 is negative (see also [4]). In the broken phase the correction is certainly not linear for small L, both in Q_M and Q_χ. Therefore we have estimated Q_0 from the two largest lattices. The sign of $a_1(\chi)$ is positive.

In Fig. 2 we show the functions F_O and A_0 for $\omega = 1.2$. For large $|x|-$values, where F_O is compatible with zero we obtain from $A_0(x)$

$$\begin{aligned} B &= 0.825(1) \, , \\ C_+ &= 0.0587(8) \, , \\ C_- &= 0.01243(12) \, . \\ C_+/C_- &= 4.72(11) \, . \end{aligned}$$

Our result for C_+/C_- is in excellent agreement with the $3d$ Ising model value 4.75(3) of [2] and the latest field theoretic value 4.79(10) of [5]. In addition we estimate the ratio for the next-to-leading amplitudes of the susceptibility to $a_{1+}/a_{1-} = -0.37(2)$. Variations of ω lead only to slight changes in the critical amplitudes. The correction-to-scaling amplitudes are however more affected. Here one should note that we describe the whole correction-to-scaling contributions with a single term. Consequently, the value of ω is somewhat higher as expected from the relation $\theta = \omega\nu = 0.51(3)$.

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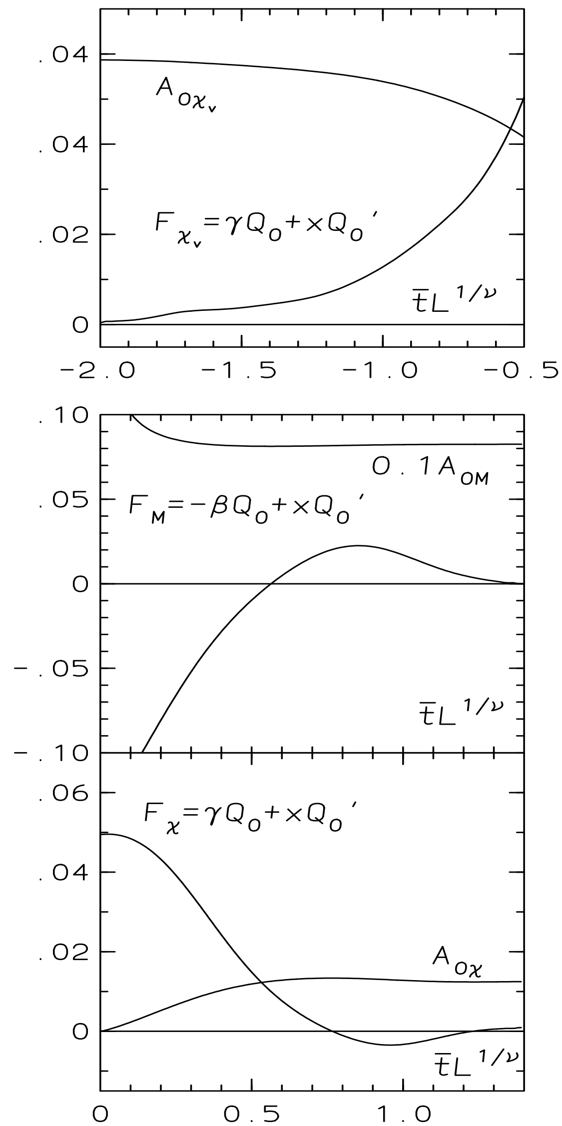

Figure 2. The control functions F_O and the amplitude functions A_0 for $\omega = 1.2$.